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STEM 2A

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Who knew Humans, Ants, and Whales all had their own Geometries?

What if someone told you that the geometry you know too well, the one you practiced on the white board millions of times in eighth grade Math, the one you swear by to this very day, *was all a lie.* Well, maybe not all of it, but you get the point – pretty big deal…

Widely-known as the father of Geometry, the Greek mathematician Euclid wrote a series of thirteen books called *Elements* in 300 BC, essentially explaining everything one would need to know about geometry. In particular, he stated The Five Postulates, which some mathematicians equate to the Five Commandments of Geometry – yes, they’re that important – well, at least for his type of geometry (You have to give the guy some credit, he even had a whole practice of geometry named after him), and these theorems are still used today to solve problems dealing with the length and height of a specific object, and often in astrology. In a nutshell, the Postulates state as follows:

1. There can be formed a straight line between any two points.
2. An indefinite straight light can be produced from any straight-line segment.
3. Given any straight light segment, a circle can be drawn with that segment as its radius and one of its endpoints on the circle.
4. All right angles are equal to one another.
5. (The Problem Child) For any point not on a line, there is exactly one line parallel to the given line.

For centuries, these axioms, along with all of his writings, were considered, for lack of a better term, true; not to be reckoned with – until some questions started developing, especially about our problem child, the Fifth Postulate, or the Parallel Postulate, which has been often used to prove that any and every triangle has an angle sum of 180 degrees. This is where people started to scratch their heads, that is, wonder if there was another type of valid geometry in which this so-called rule cannot prove to be true, and boy, were they right…

Spherical Geometry, you guessed it, is the geometry used on a sphere, like our very own Earth; and yes, sadly, the Earth was once perceived as flat, which is probably one of the reasons behind our friend Euclid’s “flat surface geometry”. But, not to dwell on the past, it was eventually learned that our Earth is indeed a spherical shape, which, in turn, lead many mathematicians to developing this “Non-Euclidean” geometry, and how it is alike and differs from Euclidean geometry. First of all (surprise!), the Fifth Postulate does *not* work in spherical geometry. In fact, for any given point on a line, there are exactly *no* lines parallel to the given line – there are no parallel lines at all! This is because every line segment on a sphere, as stated in the Second Postulate, can produce an indefinite line, which, on a sphere, creates a “great circle”, or “geodesic”, a line that goes all the way around a sphere, which always manages to bisect the sphere because its center is always the core of the sphere. But, if every geodesic *must* bisect the sphere, then, by right, every geodesic *must* intersect every geodesic (there are infinite amounts of geodesics), thus proving the Fifth Postulate completely and utterly wrong. Tragic. Other than that, the other four Postulates still manage to work in spherical geometry, but one mustn’t forget another key player in both geometries: shapes.

Yes, polygons exist in both types of geometry, but how to we define a polygon in Euclidean Geometry? A shape with a straight line that has at least three sides? Sure, but that won’t seem to cut it for spherical, and that’s why definitions are so important here. See, every time two geodesics intersect on a sphere, they actually create two two-sided polygons, unique to spherical geometry, called lunes. This intersection also ends up creating two triangles, whose angle measures will always exceed 180 degrees, but never go over 900 degrees. Triangles on a sphere also can never be similar because every time the area of a particular spherical triangle increases, it needs to cover more surface on a sphere, until eventually it reaches a great circle, for which case it must cover less surface on the sphere. This means any time the triangle changes surface area, its angles must change as well. In order for two triangles to have the same angle measure, their areas must be the same, and visa-versa. We all know the area of a sphere is four times pie times the radius squared, but who knew that the area of a sphere could also be determined using Girard’s Theorem? This rule states that the area of the lunes and triangles created by any three points on a sphere added up, minus the excess triangles that are created from the intersection of the two geodesics, divided by 720 equals the area of the sphere, which proves that knowing the area of the lunes and triangles on a sphere *can* help determine the surface area of the sphere.

This particular geometry can be seen used for navigation, in particular, determining the fastest route for a plane or ship to travel between any two points, which is by traveling along the geodesic. This geometry can be referred to as an ant’s geometry because an ant is always traveling along the ground surface of the earth, which is, in fact, spherical.

Another type of geometry that differs from Euclidean, and even Spherical Geometry, is Hyperbolic Geometry, known as the whale’s geometry because of its eternal curvature, like the ocean. Hyperbolic Geometry, invented by the Russian mathematician Nikolai Ivanoich Lobachevsky in 1829, is a geometry in which all of Euclid’s Postulates work, except for, you guessed it, the problem child Fifth Postulate. It demonstrates positive and negative curvature in a natural plane, much like a horse saddle or a Pringle Chip, or the sound waves whales use to communicate with each other under water. In this case, for any given point not on a line, there are *infinitely many* parallel lines that can be drawn through that point to the other line. This is because in this type of geometry, there are no straight edges; everything is curved, which means no two lines can ever be the same distance apart the whole way. Also, the sum of the angles of any triangle always must always be *less* than 180 degrees because the angles are so compressed since everything is curved. The Pythagorean theorem is therefore false, and there are triangles that you simply cannot draw a circle around because of these weird curvatures. Like Spherical Geometry, there are no rectangles defined by ninety-degree angles, and there are no similar triangles, but unlike Spherical Geometry, there is a fixed number in Hyperbolic Geometry such that every triangle area is less than that fixed number. Also, the ever-so important lunes on a sphere do not exist in hyperbolic geometry, and therefore all polygons in this geometry *must* have three or more sides, not two.

So, you have heard it. The math you have probably learned like the back of your hand, believe it or not, has definitely had some of its rules proven wrong. The problem child, the Fifth Postulate, cannot be used in Hyperbolic Geometry, where there are infinitely many lines parallel to any given line, or Spherical Geometry, where there are no lines parallel to any given line. Nowadays, it is probably hard for one to understand why, for so many years, Euclidean Geometry was the Holy Grail of Geometry, a geometry not to be questioned, a geometry that was simply right all just because a guy named Euclid put it down in a bunch of books. It is the people who question the rules, who think out of the box, and try to explain the unexplainable, that are the ones that make these brilliant discoveries, like Albert Einstein. Even in this day and age, there are still so many unanswered questions, not just about geometry, but about, literally, everything, like what the shape of the universe is, which it can assumed to be hyperbolic, but not proven (yet). Like Spherical Geometry and Hyperbolic Geometry, the next big breakthrough is probably on its way to come into fruition, and it is our job as the next generation to question the rules, to take risks, and most importantly, to make a difference that could positively change the path of history – forever.